

ON THE REACTION $ep \rightarrow ep\gamma$ *

M.V. Galynsky and M.I. Levchuk [†]

Institute of Physics, Belarusian Academy of Sciences, Minsk

Abstract

We have studied the reaction $ep \rightarrow ep\gamma$ in the kinematics corresponding to electron scattering at small angles and photon scattering at large angles, where proton bremsstrahlung dominates. The analysis is based on the direct evaluation method of the matrix elements in the so-called diagonal spin basis. The results of numerical calculations for electron beam energy $E_e = 200$ MeV in the above kinematics show that the relative contribution of the Bethe-Heitler and interference terms to the reaction cross section is less than 10 %, and the cross section for the reaction $ep \rightarrow ep\gamma$ is quite sensitive to the proton polarizability. Owing to the factorization of the squared electric and magnetic form factors of the proton, a compact expression has been obtained for the differential cross section of the Bethe-Heitler emission of a linearly polarized photon by an electron, taking into account the proton recoil and form factors. A covariant expression has been obtained for the lepton tensor in which contributions of states with transverse and longitudinal polarizations of the virtual photon are separated.

1 The reaction $ep \rightarrow ep\gamma$ and the proton polarizability

There has recently been much interest in studying Compton scattering on nucleons at low and intermediate energies. The motivation is that the fundamental structure constants of the nucleon, the electric and magnetic polarizabilities, can be determined in this process. The nucleon polarizabilities contain important information about the nucleon structure at large and intermediate distances, in particular, about the radius of the quark core, the meson cloud, and so on. A detailed discussion of these questions can be found in [1, 2]. Knowledge of the amplitudes for Compton scattering on nucleons is also required to interpret the data on photon scattering off nuclei. For example, such studies can answer the question of in what degree the electromagnetic properties of free and bound nucleons differ.

All the experimental results on the proton polarizabilities have been obtained from data on elastic γp scattering below pion photoproduction threshold [3]. However, it has recently been shown that measurements of the proton polarizabilities at the Novosibirsk storage ring with electron beam energy of 200 MeV using an internal jet target appear to be very promising. As proposed in [4], this can be done using the reaction

$$e^-(p_1) + p^+(q_1) \rightarrow e^-(p_2) + p^+(q_2) + \gamma(k) \quad (1)$$

in the kinematics corresponding to electron scattering at small angles and photon scattering at large angles, i.e. in conditions of small 4-momentum transfer from the initial electron to the final photon and proton. In the lowest order of perturbation theory, the process (1) is described by three graphs shown in Fig.1.

The first two (a) and (b) correspond to electron bremsstrahlung (Bethe-Heitler graphs), and the third (c) corresponds to proton bremsstrahlung (graph with virtual Compton scattering (VCS) on a proton). The kinematics described above was chosen for the following reasons. First, the subprocess of real Compton scattering (RCS) on the proton is realized in it since at small electron scattering angles the virtual photon with 4-momentum $r = p_1 - p_2$ (see Fig.1) becomes almost real. Here the quantity $|r| = \sqrt{-(p_1 - p_2)^2}$ turns out to be small, $|r| \sim m$, where m is the electron mass. Second, for electron scattering at small angles and

*Talk at The Vth International Summer School-Seminar *Actual Problems of Particle Physics* (Belarus, Gomel, July 30 - August 8 1999)

[†]E-mails: galynski@dragon.bas-net.by, levchuk@dragon.bas-net.by

photon scattering at large angles, the contribution of the graph corresponding to proton bremsstrahlung dominates, being several orders of magnitude larger than the contribution of the Bethe-Heitler graphs to the cross section for the process (1) [5]. This is the main requirement needed to separate the subprocess of Compton scattering on the proton [4] in the reaction $ep \rightarrow ep\gamma$.

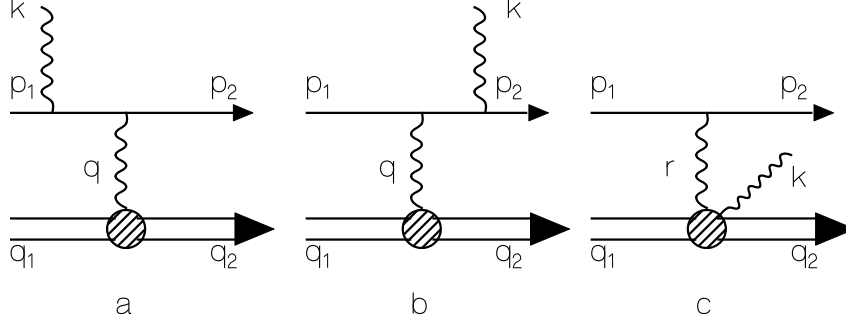


Figure 1: Graphs corresponding to the reaction $ep \rightarrow ep\gamma$.

The estimates in the framework of the method of equivalent photons for a scalar model [4] showed that the reaction (1) offers a good possibility of obtaining high-statistics data on the Compton scattering cross section and the proton polarizability. Measurement of the electric (α_p) and magnetic (β_p) polarizabilities of the proton with higher accuracy than in earlier studies is one of the most important problems to be solved by experiments in the near future [6, 7].

However, to obtain high-statistics data on the cross section for γp scattering and the proton polarizability it is essential to use a theoretical model more accurate than that of [4]. It must include both the spin properties of the particles and parameters characterizing the electromagnetic structure of the hadron. The model can be based on the result of [8], where a general calculation of the reaction $ep \rightarrow ep\gamma$ was performed. The cross section was expressed in terms of 12 form factors corresponding to the VCS subprocess on the proton (i.e., the contribution of the graph in Fig.1c) and two form factors corresponding to the Bethe-Heitler graphs.

The differential cross section for the reaction $ep \rightarrow ep\gamma$ in the above kinematics was calculated in [9]. It was expressed in terms of the six invariant amplitudes for RCS [1, 10], and also the electric and magnetic form factors of the proton [11].

The matrix element corresponding to the sum of the two Bethe-Heitler graphs (a) and (b) in Fig.1 reads

$$M_1 = \bar{u}(p_2) Q_e^\mu u(p_1) \cdot \bar{u}(q_2) \Gamma_\mu(q^2) u(q_1) \frac{1}{q^2}, \quad (2)$$

$$Q_e^\mu = \gamma^\mu \frac{\hat{p}_1 - \hat{k} + m}{-2p_1 k} \hat{e} + \hat{e} \frac{\hat{p}_2 + \hat{k} + m}{2p_2 k} \gamma^\mu, \quad (3)$$

$$\Gamma_\mu(q^2) = f_1 \gamma_\mu + \frac{\mu_p}{4M} f_2 (\hat{q} \gamma_\mu - \gamma_\mu \hat{q}), \quad (4)$$

where $u(p_i)$ and $u(q_i)$ are the bispinors of electrons and protons with 4-momenta p_i and q_i , $p_i^2 = m^2$, $q_i^2 = M^2$, $\bar{u}(p_i) u(p_i) = 2m$, $\bar{u}(q_i) u(q_i) = 2M$, ($i = 1, 2$), $\hat{k} = k_\mu \gamma^\mu$, γ^μ are the Dirac matrices, $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, $\gamma^{5+} = \gamma^5$; μ_p , f_1 , and f_2 are respectively the anomalous magnetic moment and the Dirac and Pauli form factors of the proton [11], $q = q_2 - q_1$ is the momentum transfer, e is the polarization 4-vector of a photon with momentum k , $ek = k^2 = 0$, and M is the proton mass.

In the limit of interest $|r| \sim m$, the matrix element corresponding to the graph of Fig.1c is expressed in terms of the six invariant RCS amplitudes T_i ($i = 1, 2, \dots, 6$). It has the form [8]

$$M_2 = \bar{u}(p_2) \gamma^\mu u(p_1) \cdot \bar{u}(q_2) M_{\mu\nu} e^\nu u(q_1) \frac{1}{r^2}, \quad (5)$$

$$M_{\mu\nu} = \frac{C_\mu C_\nu}{C^2} (T_1 + T_2 \hat{K}) + \frac{D_\mu D_\nu}{D^2} (T_3 + T_4 \hat{K}) + \quad (6)$$

$$+ \frac{(C_\mu D_\nu - C_\nu D_\mu)}{D^2} \gamma^5 T_5 + \frac{(C_\mu D_\nu + C_\nu D_\mu)}{D^2} T_6 \hat{D} . \quad (7)$$

The tensor $M_{\mu\nu}$ is constructed using a set of four mutually orthogonal 4-vectors C, D, B , and K :

$$\begin{aligned} K &= 1/2 (r + k) , \quad Q = 1/2(r - k) , \quad R = 1/2(q_1 + q_2) , \\ C &= R - \frac{(RK)}{K^2} K - \frac{(RB)}{B^2} B , \quad B = Q - \frac{(QK)}{K^2} K , \\ D_\mu &= \varepsilon_{\mu\nu\rho\sigma} K^\nu B^\rho C^\sigma , \end{aligned} \quad (8)$$

and it satisfies the requirements of parity conservation and gauge invariance:

$$M_{\mu\nu} k^\nu = r^\mu M_{\mu\nu} = 0 . \quad (9)$$

In the unpolarized case it is most efficient to use the standard approach [11] for calculation of the differential cross section of the process (1) together with evaluation of matrix elements in the diagonal spin basis (DSB) [12]-[15]. In the DSB, the spin 4-vectors s_1 and s_2 of particles with 4-momenta p_1 and p_2 ($s_1 p_1 = s_2 p_2 = 0, s_1^2 = s_2^2 = -1$) belong to the hyperplane formed by the 4-vectors p_1 and p_2 :

$$s_1 = - \frac{(v_1 v_2) v_1 - v_2}{\sqrt{(v_1 v_2)^2 - 1}} , \quad s_2 = \frac{(v_1 v_2) v_2 - v_1}{\sqrt{(v_1 v_2)^2 - 1}} , \quad (10)$$

where $v_1 = p_1/m_1$ and $v_2 = p_2/m_2$. To find the probability for the process (1) it is sufficient to calculate the matrix elements of the electron and proton currents

$$(J_e^{\pm\delta,\delta})_\mu = \bar{u}^{\pm\delta}(p_2) \gamma_\mu u(p_1)^\delta , \quad (11)$$

$$(J_p^{\pm\delta',\delta'})_\mu = \bar{u}^{\pm\delta'}(q_2) \Gamma_\mu(q^2) u^{\delta'}(q_1) , \quad (12)$$

and also the quantity

$$X_\mu^{\pm\delta',\delta'} = \bar{u}^{\pm\delta'}(q_2) M_{\mu\nu} e^\nu u^{\delta'}(q_1) . \quad (13)$$

The calculations give [12]-[15]:

$$(J_e^{\delta,\delta})_\mu = 2m(a_0)_\mu , \quad (J_e^{-\delta,\delta})_\mu = -2\delta y_- (a_\delta)_\mu , \quad y_- = \sqrt{-p_-^2}/2 , \quad (14)$$

$$(J_p^{\delta',\delta'})_\mu = 2g_e M(b_0)_\mu , \quad (J_p^{-\delta',\delta'})_\mu = -2\delta' y'_- g_m (b_{\delta'})_\mu , \quad y'_- = \sqrt{-q_-^2}/2 , \quad (15)$$

where

$$a_0 = p_+/\sqrt{p_+^2} , \quad a_3 = p_-/\sqrt{-p_-^2} , \quad a_2 = [a_0 \cdot a_3]^\times k/\rho , \quad a_1 = [a_0 \cdot a_3]^\times a_2 , \quad (16)$$

$$p_\pm = p_2 \pm p_1 , \quad a_{\pm\delta} = a_1 \pm i\delta a_2 , \quad \delta = \pm 1 , \quad a_2 k = 0 , \quad a_1^2 = a_2^2 = a_3^2 = -a_0^2 = -1 . \quad (17)$$

$$b_0 = q_+/\sqrt{q_+^2} , \quad b_3 = q_-/\sqrt{-q_-^2} , \quad b_2 = [b_0 \cdot b_3]^\times k/\rho' , \quad b_1 = [b_0 \cdot b_3]^\times b_2 , \quad (18)$$

$$q_\pm = q_2 \pm q_1 , \quad b_{\pm\delta'} = b_1 \pm i\delta' b_2 , \quad \delta' = \pm 1 , \quad b_2 k = 0 , \quad b_1^2 = b_2^2 = b_3^2 = -b_0^2 = -1 . \quad (19)$$

In Eqs.(16), (18) and below a dot between any two 4-vectors a and b , square parentheses and symbol " \times " stands for dyadic product of vectors (but not scalar product) $a \cdot b = (a \cdot b)_{\mu\nu} = (a)_\mu (b)_\nu$, alternating dyadic $[a \cdot b] = a \cdot b - b \cdot a$ and dual operation $[a \cdot b]^\times = ([a \cdot b]^\times)_{\mu\nu} = 1/2 \varepsilon_{\mu\nu\rho\sigma} ([a \cdot b])^{\rho\sigma} = \varepsilon_{\mu\nu\rho\sigma} (a)^\rho (b)^\sigma$, respectively, $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita symbol ($\varepsilon_{0123} = -1$); ρ and ρ' are determined from the normalization conditions (17) and (19), finally, g_e and g_m are just electric and magnetic form factors of the proton (Sachs form factors):

$$g_e = f_1 + \mu_p \frac{q^2}{4M^2} f_2 , \quad g_m = f_1 + \mu_p f_2 . \quad (20)$$

Therefore, in the DSB the matrix elements of the proton current for spin-non-flip and spin-flip transitions are expressed in terms of the electric g_e and magnetic g_m form factor, respectively (see [16]).

Once the matrix elements of the proton current (12) have been determined, the calculation of the contribution of the two Bethe-Heitler graphs reduces to the calculation of VCS on the electron [9, 14, 15]:

$$|M_1^{\pm\delta',\delta'}|^2 = \frac{1}{q^4} |\bar{u}(p_2) \left(\hat{J}_p^{\pm\delta',\delta'} \frac{\hat{p}_1 - \hat{k} + m}{-2p_1 k} \hat{e} + \hat{e} \frac{\hat{p}_2 + \hat{k} + m}{2p_2 k} \hat{J}_p^{\pm\delta',\delta'} \right) u(p_1)|^2. \quad (21)$$

Denoting the result of averaging and summing the expression $|M_1^{\pm\delta',\delta'}|^2$ over the polarizations of the initial and final particles by Y_{ee} , one obtains [9, 14, 15]:

$$Y_{ee} = 1/4 \sum_{\delta'e} Tr\{ (\hat{p}_2 + m) \hat{Q}_e^{\pm\delta',\delta'} (\hat{p}_1 + m) \hat{Q}_e^{\pm\delta',\delta'} \} / q^4, \quad (22)$$

where $\hat{Q}_e^{\pm\delta',\delta'} = (Q_e^\mu) (J_p^{\pm\delta',\delta'})_\mu$ is the operator in parentheses between the electron bispinors $\bar{u}(p_2)$ and $u(p_1)$ in Eq. (21), and $\hat{Q}_e^{\pm\delta',\delta'} = \gamma_0 (\hat{Q}_e^{\pm\delta',\delta'})^+ \gamma_0$. Owing to the factorization of the electric and magnetic form factors g_e and g_m in (15), the Bethe-Heitler term in the cross section for the reaction $ep \rightarrow ep\gamma$ Y_{ee} (22) contains only the squares of the Sachs form factors (see [9, 12, 14, 15, 17, 18]).

Similarly, the calculation of the contribution from the graph in Fig.1c reduces to the calculation of quasi-real Compton scattering on the proton. Using the expressions for the electron current (14), one has

$$|M_2^{\pm\delta,\delta}|^2 = \frac{1}{r^4} |\bar{u}(q_2) \hat{Q}_p^{\pm\delta,\delta} u(q_1)|^2, \quad (23)$$

where $\hat{Q}_p^{\pm\delta,\delta} = (J_p^{\pm\delta,\delta})^\mu M_{\mu\nu} e^\nu$. Denoting the result of averaging and summing Eq. (23) over the polarizations of the initial and final particles by Y_{pp} , we obtain [9]:

$$Y_{pp} = 1/4 \sum_{\delta e} Tr\{ (\hat{q}_2 + M) \hat{Q}_p^{\pm\delta,\delta} (\hat{q}_1 + M) \hat{Q}_p^{\pm\delta,\delta} \} / r^4, \quad (24)$$

where $\hat{Q}_p^{\pm\delta,\delta} = \gamma^0 (\hat{Q}_p^{\pm\delta,\delta})^+ \gamma^0$. Finally, to calculate the interference term in the case of unpolarized particles

$$Y_{ep} = 1/4 \sum_{\delta,\delta',e} 2Re M_1 M_2^* \quad (25)$$

we shall use the matrix elements of the proton current (15) and also the 4-vectors $X_\mu^{\pm\delta',\delta'}$ (13), which have the form [9]

$$\begin{aligned} X_\mu^{-\delta',\delta'} &= -2\delta' y'_- b_1 k \left(\frac{C_\mu C_\nu}{C^2} T_2 + \frac{D_\mu D_\nu}{D^2} T_4 + i\delta' y'_+ y'_- \frac{(C_\mu D_\nu + C_\nu D_\mu)}{D^2} T_6 \right) e^\nu, \\ X_\mu^{\delta',\delta'} &= 2 \left(y'_+ \left(\frac{C_\mu C_\nu}{C^2} \left(T_1 + \frac{\nu_1 M}{1-\tau} T_2 \right) + \frac{D_\mu D_\nu}{D^2} \left(T_3 + \frac{\nu_1 M}{1-\tau} T_4 \right) \right) + \right. \\ &\quad \left. + \delta' y'_- \frac{(C_\mu D_\nu - C_\nu D_\mu)}{D^2} T_5 \right) e^\nu, \end{aligned} \quad (26)$$

where $y'_+ = \sqrt{q_+^2}/2 = M\sqrt{1-\tau}$, $\tau = q^2/4M^2$ and $\nu_1 = kq_+/2M^2$. As a result, one has for the matrix element M_2 (5)

$$M_2 = \bar{u}(p_2) \hat{X}^{\pm\delta',\delta'} u(p_1) / r^2, \quad (27)$$

and Eq.(25) reduces to the trace [9]:

$$Y_{ep} = 1/4 \sum_{\delta, \delta', e} 2Re \{ Tr ((\hat{p}_2 + m) \hat{Q}_e^{\pm\delta', \delta'} (\hat{p}_1 + m) \hat{X}^{\pm\delta', \delta'}) \} / q^2 / r^2, \quad (28)$$

where $\hat{X}^{\pm\delta', \delta'} = \gamma^\mu X_\mu^{\pm\delta', \delta'}$ and $\hat{X}^{\pm\delta', \delta'} = (X_\mu^{\pm\delta', \delta'})^* \gamma^\mu$. The interference term Y_{ep} (28) is a linear combination of the proton electric and magnetic form factors, because the operators $\hat{Q}_e^{\pm\delta', \delta'}$ are expressed linearly in terms of the matrix elements of the proton current: $\hat{Q}_e^{\pm\delta', \delta'} = (Q_e)^\mu (J_p^{\pm\delta', \delta'})_\mu$, (see Eqs. (3) and (15)). Therefore, the problem of finding the probability for the reaction $ep \rightarrow ep\gamma$ in this approach has been reduced to calculations of the traces (22), (24), and (28), which were done making the use of the program REDUCE. For the differential cross section we then obtained [9, 14]:

$$d\sigma = \frac{\alpha^3 |T|^2 \delta^4(p_1 + q_1 - p_2 - q_2 - k)}{2\pi^2 \sqrt{(p_1 q_1)^2 - m^2 M^2}} \frac{d^3 \vec{p}_2}{2p_{20}} \frac{d^3 \vec{q}_2}{2q_{20}} \frac{d^3 \vec{k}}{2\omega}, \quad (29)$$

$$|T|^2 = 1/4 \sum_{pol} |M_{fi}|^2 = Y_{ee} + Y_{ep} + Y_{pp}, \quad (30)$$

$$Y_{ee} = \frac{8M^2}{q^4} (g_e^2 Y_I + \tau g_m^2 Y_{II}), \quad (31)$$

$$Y_I = -\frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} - \frac{m^2 q^2}{2} \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)^2 - \frac{r^2 q^2}{2\lambda_1 \lambda_2} - \frac{m^2}{2M^2(1-\tau)} \left(\frac{p_1 q_+}{\lambda_2} - \frac{p_2 q_+}{\lambda_1} \right)^2 - \frac{\tau}{(1-\tau)} \frac{((p_1 q_+)^2 + (p_2 q_+)^2)}{\lambda_1 \lambda_2}, \quad (32)$$

$$Y_{II} = -\frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} - \frac{m^2 q^2}{2} \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right)^2 - \frac{r^2 q^2}{2\lambda_1 \lambda_2} + \frac{m^2}{2M^2(1-\tau)} \left(\frac{p_1 q_+}{\lambda_2} - \frac{p_2 q_+}{\lambda_1} \right)^2 + \frac{\tau}{(1-\tau)} \frac{((p_1 q_+)^2 + (p_2 q_+)^2)}{\lambda_1 \lambda_2} - 2 \left(\frac{m^2}{\lambda_1} - \frac{m^2}{\lambda_2} \right)^2 + 4 m^2 \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right), \quad (33)$$

$$Y_{ep} = -\frac{32M^3}{r^2 q^2 (4\nu_4^2 - \nu_2^2)} \left\{ g_e Re \left[y_1 \left(T_1 + \frac{\nu_1 M}{1-\tau} T_2 \right) + y_2 \left(T_3 + \frac{\nu_1 M}{1-\tau} T_4 \right) \right] + \tau g_m \left[-\frac{\nu_1 M}{1-\tau} Re(y_1 T_2 + y_2 T_4) + 4M Re(z_1 T_2 + z_2 T_4 + z_3 T_6) \right] \right\}, \quad (34)$$

$$Y_{pp} = -\left\{ (\alpha_1^2 \alpha_3 + \nu_3) [(1-\tau) |T_1|^2 + 2\nu_1 M Re(T_1 T_2^*) + M^2 (\nu_1^2 - \nu_2^2) |T_2|^2] + (\alpha_2 + \nu_3) [(1-\tau) |T_3|^2 + 2\nu_1 M Re(T_3 T_4^*) + M^2 (\nu_1^2 - \nu_2^2) |T_4|^2] + (\alpha_1^2 \alpha_3 + \alpha_2 + 2\nu_3) \tau \left(-\frac{|T_5|^2}{M^4 \nu_2^2} + \frac{M^2}{\alpha_3} |T_6|^2 \right) \right\} \frac{16M^4}{r^4}. \quad (35)$$

For the invariant variables in Eqs. (29) – (35) used in determining the Bethe-Heitler term (Y_{ee}), the interference term (Y_{ep}), and the term corresponding to proton bremsstrahlung (Y_{pp}), we used the notation adopted in [8]:

$$\begin{aligned}
y_1 &= 2\alpha_1[\alpha_1\alpha_3(\nu_2\nu_5 - \nu_1\nu_4) + 2\nu_4^2 + \nu_2\nu_3], \quad \nu_1 = kq_+/2M^2, \quad \nu_2 = -kq_-/2M^2, \\
y_2 &= 2\alpha_2(\nu_2\nu_5 - \nu_1\nu_4) - \alpha_1\nu_2^2, \quad \nu_3 = r^2/4M^2, \quad \nu_4 = kq_+/4M^2, \quad \nu_5 = p_+q_+/4M^2, \\
y_3 &= -(4\nu_3/\nu_2^2)[\alpha_1\alpha_3(\nu_1\nu_2(\nu_2 + \nu_3) - 2\nu_4(\nu_1\nu_4 - \nu_2\nu_5)) + \nu_4(4\nu_4^2 - \nu_2^2)], \\
\alpha_1 &= \nu_5 + \nu_1\nu_4(2\nu_3 + \nu_2)/\nu_2^2, \quad \alpha_3 = \nu_2^2/(\nu_2^2 + (\nu_2 + \nu_3)(\nu_1^2 - \nu_2^2)), \\
\alpha_2 &= m^2/M^2 - \nu_3 + M^6/D^2[-(\nu_1\nu_4 + \nu_2\nu_5)^2 + 4\nu_3(\nu_4^2 - \nu_1\nu_4\nu_5) - 4\nu_3\nu_4^2(\nu_2 + \nu_3)], \\
D^2 &= M^6(\nu_2^2 + (\nu_2 + \nu_3)(\nu_1^2 - \nu_2^2)) = M^6\nu_2^2/\alpha_3, \quad \lambda_1 = p_1k, \quad \lambda_2 = p_2k, \\
z_1 &= \nu_1\nu_4\alpha_1^2\alpha_3, \quad z_2 = \nu_2\nu_4\alpha_2, \quad z_3 = 1/4\alpha_1(2\nu_2(2\alpha_2 + \nu_2 + \nu_3) + 4\nu_4^2 - \nu_2^2).
\end{aligned}$$

It should be noted that the expression obtained for the differential cross section (29) coincides, within the definition of the initial quantities (the tensor $M_{\mu\nu}$), with the result obtained in [8], if one expresses in the latter the form factors f_1 and f_2 through g_e and g_m . Nevertheless, the Bethe-Heitler term Y_{ee} and the interference term Y_{ep} have a more compact form due to the factorization of the electric and magnetic form factors.

Let us consider contributions of all three graphs to the cross section for the reaction (1) in the selected kinematics when the initial proton is at rest ($q_1 = (M, 0)$), and the electron beam energy is $E_e = 200$ MeV. Performing the required integration over the phase space we obtain [9]:

$$d\sigma = \frac{\alpha^3 \omega^2 |\vec{q}_2| |T|^2}{16\pi^2 M |\vec{p}_1| (p_2 k)} dE_{pk} d\Omega_{q_2} d\Omega_\gamma, \quad (36)$$

where $d\Omega_\gamma$ and $d\Omega_{q_2}$ are the elements of the photon and proton solid angles, and E_{pk} is the kinetic energy of the recoil proton. The differential cross section (36) was calculated numerically in the region $5 \leq E_{pk} \leq 35$ MeV with the sum and the difference of the electric (α_p) and magnetic (β_p) polarizabilities equal to $\alpha_p + \beta_p = 14$ and $\alpha_p - \beta_p = 10$ (in units of $10^{-4} fm^3$) [1]–[4]. We assume that the reaction kinematics is planar, and that the photon emission and proton scattering angles are $\vartheta_\gamma = 135^\circ$ and $\vartheta_p = -20.5^\circ$, respectively (all angles are measured from the direction of the primary electron beam). Calculations show that in the entire range of proton kinetic energy considered, $5 \leq E_{pk} \leq 35$ MeV, for the selected angles $\vartheta_\gamma = 135^\circ$ and $\vartheta_p = -20.5^\circ$, the electron scattering angle ϑ_e and the 4-momentum transfer $|r| = \sqrt{-(p_2 - p_1)^2}$ are bounded by the values $|\vartheta_e| \leq 6.4^\circ$ and $|r| \leq 7.3$ MeV, with the minimum value of $|r|$ corresponding to forward electron scattering.

The results of numerical calculations of the cross section (36), $d\sigma/dE_{pk}/d\Omega_{q_2}/d\Omega_\gamma$ in the above kinematics are shown in Fig.2. We see that in the angular range studied the cross section for the reaction $ep \rightarrow ep\gamma$ has a sharp peak consisting of two maxima. This peak originates from the factor $1/r^4$ in Eq. (35) for Y_{pp} . The two maxima have a kinematical origin and arise from the interference of two pole graphs corresponding to quasi-real Compton scattering. The cross section (36) has a strong angular dependence, which, in particular, causes the two maxima to disappear when the proton scattered (or photon emission) angle is changed by only one degree (i.e., for $\vartheta_p = -19.5^\circ$), so that we have an ordinary peak at $E_{pk} = 25$ MeV.

The differential cross section (36) shown in Fig.2, is the sum of the Bethe-Heitler (σ_{ee}), the interference (σ_{ep}), and the proton (σ_{pp}) terms (see (30)), where the symbol (σ) denotes the cross section of the form (36) with $|T|^2$ replaced by Y_{ee} , Y_{ep} , and Y_{pp} , respectively. Numerical calculations show that in the entire range of proton kinetic energy studied, $5 \leq E_{pk} \leq 35$ MeV, the ratios of the Bethe-Heitler term σ_{ee} and the interference term σ_{ep} to the term corresponding to proton emission σ_{pp} are bounded by the values $\sigma_{ee}/\sigma_{pp} < 0.02$ and $|\sigma_{ep}|/\sigma_{pp} < 0.05$. The calculations carried out for another set of angles ($\vartheta_\gamma = 135^\circ$ and $\vartheta_p = -20^\circ$) give results which are only slightly different: $\sigma_{ee}/\sigma_{pp} < 0.05$ and $|\sigma_{ep}|/\sigma_{pp} < 0.075$. Since these ratios are much smaller than unity, the main requirement (see [4]) for separation of the background, which is mainly electron bremsstrahlung, is satisfied.

To investigate the sensitivity of the reaction (1) to the proton polarizability we performed numerical calculations of the cross section (36) for the same set of angles ($\vartheta_\gamma = 135^\circ$ and $\vartheta_p = -20^\circ$) and fixed sum of the electric and magnetic polarizabilities $\alpha_p + \beta_p = 14$ but different values of the difference: (a) $\alpha_p - \beta_p = 10$

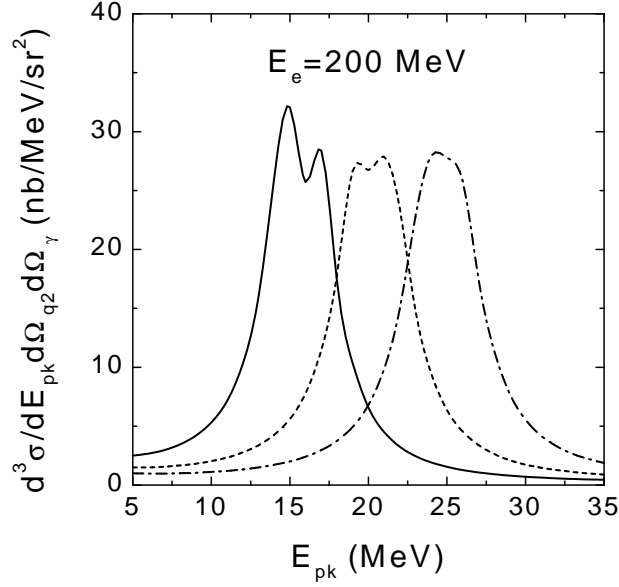


Figure 2: Differential cross section (36) for the reaction (1) in the kinematics where proton bremsstrahlung dominates (see comments in the text). Proton scattering angles are $\vartheta_p = -20.5^\circ$ (solid line), $\vartheta_p = -20.0^\circ$ (dashed line), $\vartheta_p = -19.5^\circ$ (dot-dashed line) and photon emission angle is $\vartheta_\gamma = 135^\circ$.

and (b) $\alpha_p - \beta_p = 6$. It turned out that the cross section (36) is about 8% larger for the smaller difference of polarizabilities. Therefore, in this kinematics the cross section for the reaction $ep \rightarrow ep\gamma$ is quite sensitive to the proton polarizability [9].

2 Emission of a linearly polarized photon by an electron in the reaction $ep \rightarrow ep\gamma$

Let us consider now the emission of a linearly polarized photon by an electron in the reaction $ep \rightarrow ep\gamma$, taking into account the proton recoil and form factors. Our study will be limited to the contribution of the two Bethe-Heitler graphs (a) and (b) in Fig.1, which corresponds to the matrix element (2). The contribution of the graph with VCS on a proton can be neglected when the initial electrons have ultrarelativistic energies, and the photon and final electron are scattered at small forward angles ($\vartheta_\gamma \sim m/E_e$, $\vartheta_e \sim m/E_e$, $m/E_e \ll 1$).

We are interested in these effects for the following reasons. First, even though the Bethe-Heitler process has been intensively studied earlier in the case of the emission of linearly polarized photons [19, 20] and is widely used to obtain them at accelerators [21], up to now the proton recoil and form factors have not been accurately taken into account (in contrast to the unpolarized case). Second, as was shown in [22], the inclusion of these factors in the case of unpolarized photons leads to a strong change of the differential cross section for the Bethe-Heitler process. Since the polarization characteristic of the scattered radiation are expressed in terms of the differential cross section for the emission of an unpolarized photon (see below), it is clear that inclusion of the recoil and form factors is essential.

The covariant expression for the differential cross section for the Bethe-Heitler process (in the Born approximation) taking into account the proton recoil and form factors in the case of emission of a linearly polarized photon has been obtained by us in [23]. It has the form

$$d\sigma_{BH} = \frac{\alpha^3 |T_e|^2 \delta^4(p_1 + q_1 - p_2 - q_2 - k)}{2\pi^2 \sqrt{(p_1 q_1)^2 - m^2 M^2}} \frac{d^3 \vec{p}_2}{2p_{20}} \frac{d^3 \vec{q}_2}{2q_{20}} \frac{d^3 \vec{k}}{2\omega}, \quad (37)$$

$$|T_e|^2 = \frac{4M^2}{q^4} (g_e^2 Y_I^e + \tau g_m^2 Y_{II}^e), \quad (38)$$

$$Y_I^e = 2 - \frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} - \frac{\tau}{1-\tau} \frac{(kq_+)^2}{\lambda_1 \lambda_2} + q^2 (ea)^2 + 4 (eA)^2, \quad (39)$$

$$Y_{II}^e = -2 - \frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} + \frac{\tau}{1-\tau} \frac{(kq_+)^2}{\lambda_1 \lambda_2} + (q^2 + 4m^2) (ea)^2 - 4 (eA)^2, \quad (40)$$

$$a = \frac{p_1}{\lambda_1} - \frac{p_2}{\lambda_2}, \quad A = b_0 + \frac{(b_0 p_2) p_1}{\lambda_1} - \frac{(b_0 p_1) p_2}{\lambda_2}. \quad (41)$$

All the quantities entering (37)-(41) are defined in the previous section. Thus, the differential cross section for the Bethe-Heitler process in the case of emission of a linearly polarized photon $d\sigma_{BH}$ (37) is naturally splitted into the sum of two terms containing only the squares of the Sachs form factors and corresponding to the contribution of transitions without ($\sim g_e^2 Y_I^e$) and with ($\sim \tau g_m^2 Y_{II}^e$) proton spin flip.

Let us discuss the properties of the 4-vector a , which is well known from the theory of emission of long-wavelength photons [11], and the 4-vector A . They both satisfy a condition which follows naturally from the requirement of gauge invariance: $a \cdot k = A \cdot k = 0$, and, in addition, they are spacelike vectors: $a^2 < 0$ and $A^2 < 0$. This is easily verified by using the 4-momentum conservation law and the explicit form of a^2 and A^2 :

$$a^2 = m^2 \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)^2 + \frac{r^2}{\lambda_1 \lambda_2},$$

$$A^2 = 1 + \frac{m^2}{4M^2(1-\tau)} \left(\frac{q+p_1}{\lambda_2} - \frac{q+p_2}{\lambda_1} \right) + \frac{\tau}{1-\tau} \frac{q+p_1 \cdot q+p_2}{\lambda_1 \lambda_2}.$$

We note that the 4-vector A was first introduced in [23].

Using the electron 4-momenta p_1 and p_2 and the photon 4-momenta k , we construct the 4-vectors of the photon linear polarization e_{\parallel} and e_{\perp} ($e_{\parallel} \cdot k = e_{\perp} \cdot k = e_{\parallel} \cdot e_{\perp} = 0$):

$$e_{\parallel} = \frac{(p_2 k) p_1 - (p_1 k) p_2}{\rho'}, \quad e_{\perp} = \frac{[p_1 \cdot p_2]^{\times} k}{\rho'},$$

where ρ' is determined from the normalization conditions: $e_{\parallel}^2 = e_{\perp}^2 = -1$. Then the degree of photon linear polarization will be given by the following expressions [23]:

$$P_{\gamma} = \frac{|T_{\perp}|^2 - |T_{\parallel}|^2}{|T_{\perp}|^2 + |T_{\parallel}|^2} = \frac{A_1}{A_2}, \quad (42)$$

where

$$A_1 = \frac{16 M^2}{q^4} (g_e^2 A_{11} + \tau g_m^2 A_{12}), \quad (43)$$

$$A_2 = \frac{8 M^2}{q^4} (g_e^2 Y_1 + \tau g_m^2 Y_2), \quad (44)$$

$$Y_1 = 2 - \frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} - \frac{\tau}{1-\tau} \frac{(kq_+)^2}{\lambda_1 \lambda_2} - 2 \tau M^2 a^2 - 2 A^2, \quad (45)$$

$$Y_2 = -2 - \frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} + \frac{\tau}{1-\tau} \frac{(kq_+)^2}{\lambda_1 \lambda_2} - 2 \tau M^2 a^2 + 2 A^2 - 2 m^2 a^2. \quad (46)$$

$$A_{11} = A^2 + \tau M^2 a^2 + 2(e_{\perp} b_0)^2,$$

$$A_{12} = -A^2 + \tau M^2 a^2 - 2(e_{\perp} b_0)^2 + m^2 a^2,$$

$$(e_{\perp} b_0)^2 = - \frac{4(SD)^2}{M^2(1-\tau)a^2\lambda_1^2\lambda_2^2},$$

$$SD = 1/2 \epsilon_{\mu\nu\rho\sigma} (p_1)^{\mu} (p_2)^{\nu} (q_1)^{\rho} (q_2)^{\sigma},$$

It is easy to check that A_2 (44) coincides with the expression for Y_{ee} (31) determining the Bethe-Heitler cross section in the case of unpolarized particles: $A_2 = Y_{ee}$, and also that $Y_1 = Y_I$ and $Y_2 = Y_{II}$ (see (32) and (33)).

Therefore, owing to the factorization of the squared form factors g_e and g_m and also the use of the 4-vectors a and A (41), the differential cross section for the Bethe-Heitler process both for linearly polarized photon (38) and unpolarized photon (44), (31), can be written in a rather compact form.

An integration of Eq. (37) over $d^3\vec{q}_2$ and dp_{20} in the rest frame of the initial proton ($q_1 = (M, 0)$) gives the following result:

$$\frac{d\sigma_{BH}}{d\omega d\Omega_\gamma d\Omega_e} = \frac{\alpha^3 \omega}{(2\pi)^2} \frac{|\vec{p}_2|}{|\vec{p}_1|} \frac{|T|^2}{q^4}, \quad (47)$$

$$|T|^2 = g_e^2 Y_I^e + \tau g_m^2 Y_{II}^e. \quad (48)$$

Let us consider the limit of the cross section (47) when the proton is a pointlike (structureless) particle with infinite mass, i.e., we assume that $g_e = g_m = 1$ and $q_2 = (M, \vec{q}) \simeq (M, 0)$, where $\vec{q} = \vec{p}_1 - \vec{p}_2 - \vec{k}$ is the momentum transferred to the proton. In this limit ($M \rightarrow \infty$), $E_{kp} = \vec{q}^2/2M \rightarrow 0$, $\vec{q}/2M \rightarrow 0$, and $b_0 = (1, \vec{q}/2M) \simeq (1, 0)$. We choose the Coulomb gauge for the photon polarization vectors: $e = (0, \vec{e})$ in which one obtains

$$eb_0 = 0, \quad ea = \frac{p_1 e}{\lambda_1} - \frac{p_2 e}{\lambda_2}, \quad eA = p_{20} \frac{p_1 e}{\lambda_1} - p_{10} \frac{p_2 e}{\lambda_2}, \quad \tau(q+k)^2 = \omega^2 q^2.$$

Using these expressions we have in the above limit for (48):

$$|T|^2 = 2 - \frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} - \frac{\omega^2 q^2}{\lambda_1 \lambda_2} + q^2 (ea)^2 + 4 (eA)^2, \quad (49)$$

or, in expanded form,

$$\begin{aligned} |T|^2 = & 2 - \frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} - \frac{\omega^2 q^2}{\lambda_1 \lambda_2} + (4p_{20}^2 + q^2) \left(\frac{p_1 e}{\lambda_1} \right)^2 \\ & + (4p_{10}^2 + q^2) \left(\frac{p_2 e}{\lambda_2} \right)^2 - 2(4p_{10}p_{20} + q^2) \frac{p_1 e \cdot p_2 e}{\lambda_1 \lambda_2}. \end{aligned} \quad (50)$$

The expressions (47), (50) for the differential cross section for the Bethe-Heitler process $d\sigma_{BH}/d\omega/d\Omega_\gamma/d\Omega_e$ in the limit where the proton is an infinitely heavy, structureless particle coincide with the result of [19].

3 Virtual-photon polarization in the reaction

$$ep \rightarrow ep\gamma \quad (ep \rightarrow eX)$$

The reactions $ep \rightarrow ep\gamma$ and VCS on the proton have recently become interesting not only at low and intermediate energies [4], but also at high electron energies and 4-momenta transferred to the proton [7], [25]-[28]. The VCS offers greater possibilities for studying hadronic structure than the RCS process, because in it the energy and three-momentum transferred to the target can be varied independently. These attractive properties of VCS have led to the suggestion that it could be used for experimental study of the nucleon structure [25, 26] and have made it necessary to perform a thorough theoretical study of the reaction $ep \rightarrow ep\gamma$ (see [7, 27, 28] and references therein). To calculate VCS on the proton, it is necessary to know the hadron ($W_{\mu\nu}$) and lepton ($L_{\mu\nu}$) tensors [7, 29]:

$$L_{\mu\nu} = J_\mu J_\nu^*, \quad J_\mu = \bar{u}(p_2) \gamma_\mu u(p_1), \quad (51)$$

where $u(p_i)$ are electron bispinors, $\bar{u}(p_i)u(p_i) = 2m$, and m is the electron mass ($i = 1, 2$). The interpretation of the results is considerably simplified if the tensor $L_{\mu\nu}$ is expressed in terms of the longitudinal and transverse polarization vectors of the virtual photon. The corresponding expressions can be found in [7]

and [29]. However, they have two defects: (1) the electron mass is neglected, which is of course justified at ultrarelativistic electron energies and large squared 4-momentum of the virtual photon; (2) they have a noncovariant form. A lepton tensor free of these defects was constructed in [24].

Let us consider the question of the polarization state of a virtual photon with 4-momentum $r = p_1 - p_2$ which is exchanged between the electron and proton in the reaction $ep \rightarrow ep\gamma$ (see Fig.1c). Using the vectors of the orthonormal basis a_A ($A = (0, 1, 2, 3)$):

$$a_0 = p_+/\sqrt{p_+^2}, \quad a_3 = p_-/\sqrt{-p_-^2}, \quad a_2 = [a_0 \cdot a_3]^\times q_1/\rho, \quad a_1 = [a_0 \cdot a_3]^\times a_2, \quad (52)$$

$$p_\pm = p_2 \pm p_1, \quad a_2 q_1 = 0, \quad a_1^2 = a_2^2 = a_3^2 = -a_0^2 = -1,$$

which satisfies the completeness relation

$$a_0 \cdot a_0 - a_1 \cdot a_1 - a_2 \cdot a_2 - a_3 \cdot a_3 = g, \quad (53)$$

where $g = (g_{\mu\nu})$ is the metric tensor with signature $g_{\mu\nu} = (+ - - -)$, we construct the 4-vectors of the longitudinal (e_3) and transverse (e_1, e_2) polarization of a virtual photon with 4-momentum r [24]:

$$e_1 = \frac{[a_0 \cdot a_1]q_1}{\sqrt{(a_3 q_1)^2 + q_1^2}}, \quad e_2 = a_2 = \frac{[a_0 \cdot a_3]^\times q_1}{\rho}, \quad e_3 = \frac{(1 + a_3 \cdot a_3)q_1}{\sqrt{(a_3 q_1)^2 + q_1^2}}, \quad (54)$$

where

$$\rho^2 = (a_1 q_1)^2 = \frac{2p_1 p_2 \cdot p_1 q_1 \cdot p_2 q_1 - M^2((p_1 p_2)^2 - m^4) - m^2((p_1 q_1)^2 + (p_2 q_1)^2)}{(p_1 p_2)^2 - m^4}.$$

It is easily verified that the 4-vectors e_i ($i = 1, 2, 3$) are orthogonal to each other ($e_i e_j = 0, i \neq j$), and also that $e_i r = e_i a_3 = 0$ and $e_1^2 = e_2^2 = -e_3^2 = -1$. The 4-vectors e_i (54) are not changed when the auxiliary 4-vector q_1 is replaced by $q_1 + p_1 - p_2 = q_2 + k$ (since $p_1 - p_2 = r = -2ya_3$, where $y = \sqrt{-r^2}/2$, and the vectors a_A (52) are orthogonal). For this reason, the virtual-photon polarization vectors e_i (54) in the rest frame of the incident proton or in the c.m. frame of the final proton and photon can be considered as equivalent and their use lead to the same expressions. Below we restrict ourselves to the rest frame of the incident proton, $q_1 = (M, 0, 0, 0)$, where the 4-vectors e_i have the form:

$$e_1 = (0, 1, 0, 0), \quad e_2 = (0, 0, 1, 0), \quad e_3 = \frac{1}{\sqrt{-r^2}}(|\vec{r}|, r_0 \vec{n}_3), \quad (55)$$

where \vec{n}_3 is a unit vector directed along \vec{r} ($\vec{n}_3^2 = 1$), and r_0 is the time component of the 4-vector $r = (r_0, \vec{r})$.

The four mutually orthogonal vectors e_1, e_2, e_3 , and a_3 also satisfy the completeness relation:

$$e_3 \cdot e_3 - e_1 \cdot e_1 - e_2 \cdot e_2 - a_3 \cdot a_3 = g, \quad (56)$$

which allows a_0 and a_1 to be expressed in terms of e_1 and e_3 :

$$a_1 = \alpha e_3 - \beta e_1, \quad a_0 = \beta e_3 - \alpha e_1, \quad \beta^2 = 1 + \alpha^2, \quad (57)$$

$$\alpha = e_3 a_1 = a_0 e_1 = \frac{a_1 q_1}{\sqrt{(a_3 q_1)^2 + q_1^2}}, \quad \beta = e_1 a_1 = e_3 a_0 = \frac{a_0 q_1}{\sqrt{(a_3 q_1)^2 + q_1^2}}. \quad (58)$$

In the DSB (10) the matrix elements of the electron current have the form of (14). Let us write them in terms of the 4-vectors e_i (54) [24]:

$$(J_e^{\delta, \delta})_\mu = 2m (\beta e_3 - \alpha e_1)_\mu, \quad (J_e^{-\delta, \delta})_\mu = -2\delta y (\alpha e_3 - \beta e_1 + i\delta e_2)_\mu. \quad (59)$$

Therefore, for spin-non-flip transitions ($J_e^{\delta, \delta}$) the virtual-photon polarization vector is a superposition of the longitudinal (βe_3) and transverse linear ($-\alpha e_1$) polarizations, while for spin-flip transitions ($J_e^{-\delta, \delta}$) it is a superposition of the longitudinal (αe_3) and transverse elliptical [$e_\delta = (0, \vec{e}_\delta) = -\beta e_1 + i\delta e_2$] polarizations.

Here the state of a photon with elliptical polarization vector $e_\delta = (0, \vec{e}_\delta)$ has degree of linear polarization (equal to the ratio of the difference and sum of the squared semiaxes) [24]:

$$\kappa_\gamma = \frac{\beta^2 - 1}{\beta^2 + 1} = \frac{\alpha^2}{\beta^2 + 1} . \quad (60)$$

Inverting this relation, we obtain:

$$\beta^2 = \frac{1 + \kappa_\gamma}{1 - \kappa_\gamma} , \quad \alpha^2 = \frac{2\kappa_\gamma}{1 - \kappa_\gamma} .$$

Now we find the squared moduli of the vectors \vec{e}_δ and \vec{a}_δ :

$$|\vec{e}_\delta|^2 = 1 + \beta^2 = \frac{2}{1 - \kappa_\gamma} , \quad |\vec{a}_\delta|^2 = (1 + \beta^2)(1 + \kappa_L) ,$$

$$\kappa_L = \kappa_\gamma \vec{e}_3^2 = \kappa_\gamma \frac{r_0^2}{(-r^2)} , \quad \vec{e}_3^2 = \frac{r_0^2}{(-r^2)} . \quad (61)$$

Let us introduce the normalized vectors \vec{e}_δ' and \vec{a}_δ' :

$$\vec{e}_\delta' = \frac{\vec{e}_\delta}{\sqrt{1 + \beta^2}} = \sqrt{\frac{1 - \kappa_\gamma}{2}} \vec{e}_\delta , \quad |\vec{e}_\delta'|^2 = 1 . \quad (62)$$

$$\vec{a}_\delta' = \frac{\vec{a}_\delta}{\sqrt{1 + \beta^2}} = \sqrt{\frac{1 - \kappa_\gamma}{2}} \vec{a}_\delta , \quad |\vec{a}_\delta'|^2 = 1 + \kappa_\gamma \vec{e}_3^2 = 1 + \kappa_L , \quad (63)$$

It is seen that the elliptical-polarization vector \vec{e}_δ of a virtual photon can be normalized to unity ($|\vec{e}_\delta'|^2 = 1$), but the presence of a longitudinal polarization makes this normalization impossible for the total vector \vec{a}_δ' simultaneously. The quantity κ_L (61) corresponding to the inequality $|\vec{a}_\delta'|^2 = 1 + \kappa_L \neq 1$ has the meaning of the degree of longitudinal polarization of a virtual photon emitted in a transition with electron spin flip. In the ultrarelativistic limit, when the electron mass can be neglected, the quantities κ_γ and κ_L can be interpreted as the total degrees of linear and longitudinal polarization of the virtual photon. In this (massless) case we have:

$$(a_3 q_1)^2 + q_1^2 = -M^2 \frac{\vec{r}^2}{r^2} , \quad (a_1 q_1)^2 = M^2 ct g^2 \vartheta / 2 , \quad (64)$$

$$\kappa_\gamma^{-1} = 1 - 2 \frac{\vec{r}^2}{r^2} t g^2 \vartheta / 2 , \quad (65)$$

where ϑ is the angle between the vectors \vec{p}_1 and \vec{p}_2 . Equation (65) for κ_γ coincides with the result of [29].

The vector \vec{a}_δ' (63) can also be written as

$$\vec{a}_\delta' = \sqrt{\kappa_L} \vec{n}_3 - \sqrt{\frac{1 + \kappa_\gamma}{2}} \vec{e}_1 + i\delta \sqrt{\frac{1 - \kappa_\gamma}{2}} \vec{e}_2 ,$$

which makes it easy to construct the polarization density matrix for a virtual photon in the massless limit (both in the polarized case, which for massless particles is helical polarization, and in the unpolarized case; see [29]).

To obtain the complete expression for κ_γ and κ_L arising from the contributions of the matrix elements both without and with spin flip, we construct the lepton tensor averaged over electron spin states. Using the matrix elements (14) this can be easily done [24]:

$$\overline{L}_{\mu\nu} = 4m^2 (a_0)_\mu (a_0)_\nu + 4y^2 ((a_1)_\mu (a_1)_\nu + (a_2)_\mu (a_2)_\nu) . \quad (66)$$

Using the completeness condition (53) and gauge invariance, the tensor $\overline{L}_{\mu\nu}$ can be written as

$$\overline{L}_{\mu\nu} = 4x^2 (a_0)_\mu (a_0)_\nu - 4y^2 g_{\mu\nu} , \quad (67)$$

where $x^2 = m^2 + y^2$. The tensor $\bar{L}_{\mu\nu}$ (67) is used to reduce the calculation of the contribution of graphs with VCS on a proton to the cross section for the reaction $ep \rightarrow ep\gamma$ to calculation of the trace of a product of tensors:

$$Y_{pp} = \bar{L}_{\mu\nu} W_{\mu\nu}, \quad W_{\mu\nu} = V_\mu V_\nu^*, \quad V_\mu = \bar{u}(q_2) M_{\mu\nu} e^\nu u(q_1) \frac{1}{r^2}. \quad (68)$$

Let us express the tensor $\bar{L}_{\mu\nu}$ (66) in the terms of the virtual-photon polarization vectors e_i (54). As a result, it naturally breaks up into the sum of three terms corresponding to the contributions of transverse (L_T) and longitudinal (L_L) states and their interference (L_{LT}) [24]:

$$\bar{L} = 4y^2 (L_T + L_L + L_{LT}), \quad (69)$$

$$L_T = e_1 \cdot e_1 (\beta^2 + \alpha^2 m^2 / y^2) + e_2 \cdot e_2, \quad (70)$$

$$L_L = e_3 \cdot e_3 (\alpha^2 + \beta^2 m^2 / y^2), \quad (71)$$

$$L_{LT} = - (e_1 \cdot e_3 + e_3 \cdot e_1) \alpha \beta (1 + m^2 / y^2). \quad (72)$$

Then the total degree of linear polarization of the virtual photon is given by

$$\kappa'_\gamma = \frac{\beta^2 + \alpha^2 m^2 / y^2 - 1}{\beta^2 + \alpha^2 m^2 / y^2 + 1} = \frac{\alpha^2}{\beta^2 + 1 - 2m^2 / x^2}. \quad (73)$$

Since α and β are the same in Eqs. (60) and (73) (see (58)), the inclusion of the electron mass in the ultrarelativistic limit leads only to a slight increase of κ_γ [24]:

$$\kappa'_\gamma \simeq \kappa_\gamma \left(1 + \frac{2m^2}{x^2(1 + \beta^2)} \right). \quad (74)$$

Inverting the relation in (73), we find

$$\beta^2 + \alpha^2 m^2 / y^2 = \frac{1 + \kappa'_\gamma}{1 - \kappa'_\gamma}, \quad \alpha^2 + \beta^2 m^2 / y^2 = \frac{2\kappa'_\gamma}{1 - \kappa'_\gamma} + \frac{m^2}{y^2}. \quad (75)$$

We can separate the completely polarized and unpolarized parts in the transverse tensor:

$$L_T = e_1 \cdot e_1 (\beta^2 + \alpha^2 m^2 / y^2 - 1) + e_1 \cdot e_1 + e_2 \cdot e_2 = \frac{2}{1 - \kappa'_\gamma} (\kappa'_\gamma e_1 \cdot e_1 + (1 - \kappa'_\gamma) (e_1 \cdot e_1 + e_2 \cdot e_2) / 2).$$

Therefore, the virtual-photon polarization density matrix ρ_{ij} is obtained from the tensor \bar{L}_{ij} (69) just as in the massless case (see [29]):

$$\rho_{ij} = (1 - \kappa'_\gamma) \bar{L}_{ij} / 8y^2. \quad (76)$$

For the degree of longitudinal polarization of the virtual photon we then obtain:

$$\kappa'_L = \frac{r_0^2}{(-r^2)} \kappa'_\gamma \left(1 + \frac{m^2}{y^2} \frac{(1 - \kappa'_\gamma)}{2\kappa'_\gamma} \right). \quad (77)$$

The expressions (73) and (77) for κ'_γ and κ'_L with $m = 0$ obviously become κ_γ and κ_L of (60) and (61).

We conclude by noting that the region of applicability of the tensor $\bar{L}_{\mu\nu}$ (69) is not limited to only VCS on the proton. Since in fixed-target experiments the charged-lepton scattering at available energies is mainly determined by virtual photon exchange, the tensor $\bar{L}_{\mu\nu}$ (69) can also be used to study deep-inelastic electron scattering ($e^\pm p \rightarrow e^\pm X$), and muon scattering ($\mu^\pm p \rightarrow \mu^\pm X$), where inclusion of the mass is more important.

Conclusion

We have studied the reaction $ep \rightarrow ep\gamma$ in the kinematics corresponding to electron scattering at small angles and photon scattering at fairly large angles, where proton bremsstrahlung dominates. The results of numerical calculations performed in the rest frame of the initial proton at electron beam energy $E_e = 200$ MeV in the chosen kinematics show that the conditions needed to separate the subprocess $\gamma p \rightarrow \gamma p$ from the reaction $ep \rightarrow ep\gamma$ are satisfied, because the relative contribution of the Bethe-Heitler and interference terms to the reaction cross section is less than 10 %, and the cross section for the reaction $ep \rightarrow ep\gamma$ is quite sensitive to the proton polarizability.

A compact expression was obtained for the differential cross section of the Bethe-Heitler emission of a linearly polarized photon by an electron, taking into account the proton recoil and form factors, owing to the factorization of the squared electric and magnetic form factors of the proton. In the limit where the proton is a pointlike particle of infinite mass, this expression becomes to be the well-known one.

A covariant expression has been obtained for the lepton tensor in which the contribution of states with transverse and longitudinal polarization of the virtual photon is separated. It has been shown that inclusion of the lepton mass tends to increase the degree of linear polarization of the virtual photon.

Acknowledgements

The authors thank to A.I. L'vov for supplying them with a computer code for numerical calculations of the proton RCS amplitudes and for useful discussions. We are also indebted to V.A. Petrun'kin for stimulating discussions of the results.

References

- [1] V.A. Petrun'kin, Fiz. Elem. Chast. At. Yadra, **12**, (1981) 692; Sov.J.Part.Nucl. **12**, (1981) 278.
- [2] A.I. L'vov and V.A. Petrun'kin, Lecture Notes in Physics, **365**, (1990) 123.
- [3] B.E. MacGibbon et al., Phys. Rev. **C52**, (1995) 2097.
- [4] A.I. L'vov, V.A. Petrun'kin, S.G. Popov, and B.B. Wojtsekhowski, Preprint No 91-24, Budker-INP (1991); Scanned images of a preprint received at KEK library: 9108320.
- [5] P.S. Isaev and I.S. Zlatev, Nucl. Phys. **16**, (1960) 608.
- [6] B.B. Wojtsekhovski, A.I. L'vov et al., "Project: Moscow- Novosibirsk-Gottingen", Preprint Lebedev Physical Institute, Moscow (1992).
- [7] P. Kroll, M. Schurmann and P.A.M. Guichon, Nucl. Phys. **A591**, (1995) 606.
- [8] R.A. Berg and C.N. Lindner, Nucl. Phys. **26**, (1961) 259.
- [9] M.V. Galynsky, Preprint 695, IF AN BSSR, Minsk, (1994) (in Russian).
- [10] A.I. L'vov, Yad. Fiz. **34**, (1981) 1075.
- [11] V.B. Berestetskii, E.M. Lifshitz, and L.P. Pitaevskii, Quantum Electrodynamics, 2nd ed. (Pergamon Press, Oxford, 1982); Russ: original, Nauka, Moscow, 1989.
- [12] S.M. Sikach, Vestsi Akad. Nauk BSSR, Ser. Fiz.-Mat. Nauk, No 2, (1984) 84 (in Russian).
- [13] M.V. Galynsky, L.F. Zhirkov, S.M. Sikach, and F.I. Fedorov, Zh.Eksp.Teor.Fiz. **95** (1989) 1921; Sov.Phys.JETP. **68**, (1989) 1111.

- [14] M.V. Galynsky and S.M. Sikach, Phys.Part.Nucl.**29** (1998) 469; Fiz.Elem.Chast. Atom.Yadra **29** (1998) 1133; e-print hep-ph/9910284.
- [15] M.V. Galynsky and S.M. Sikach, Yad. Fiz. **54**, (1991) 1026.
- [16] F. Halsen and A.D. Martin, Quarks and Leptons: an Introductory Course in Modern Particle Physics (Wiley, New York, 1984); Russ: transl., Mir, Moscow, 1987.
- [17] A.A. Akhundov, et al., Sov. J. Nucl. Phys **44**, (1986) 988.
- [18] A.A. Akhundov, D.Yu. Bardin D.Yu. et al., Z. Phys. **C45**, (1990) 645.
- [19] R.L. Gluckstern , M.H. Hull, and G. Breit, Phys. Rev. **90**, (1953) 1026.
- [20] H. Olsen and L.C. Maximon, Phys. Rev. **114**, (1959) 887.
- [21] J. Asai, H.S. Caplan, and L.C. Maximon, Can. J. Phys. **66**, (1988) 1079.
- [22] P.S. Isaev and I.S. Zlatev, Nuovo Cim. **13**, (1959) 1.
- [23] M.V. Galynsky, Yad.Fiz.**58**, (1995) 701; Phys. Atom. Nucl. **58**, (1995) 644.
- [24] M.V. Galynsky and M.I. Levchuk, Phys.Atom.Nucl. **60**, (1997) 1855; Yad.Fiz. **60**, (1997) 2028.
- [25] C. Audit et.al., CEBAF proposal PR 93-050 (1993).
- [26] J.F.J. Van den Brand, CEBAF proposal PR 94-011 (1994).
- [27] S. Scherer, A.Yu. Korchin, and J.H. Koch, Report MKPH-T-96-4, Mainz (1996).
- [28] H.W. Fearing and S. Scherer, Report TRI-PP-96-28, MKPH-T-96-18, Mainz (1996).
- [29] A.I. Akhiezer and M.P. Rekalo, Electrodynamics of Hadrons (Naukova Dumka, Kiev, 1977, in Russian).